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A-LEVEL

# Mathematics

Mechanics 2B – MM2B

Mark scheme

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6360

June 2015

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Version/Stage: Version 1.0: Final

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Mark schemes are prepared by the Lead Assessment Writer and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all associates participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the students' responses to questions and that every associate understands and applies it in the same correct way. As preparation for standardisation each associate analyses a number of students' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, associates encounter unusual answers which have not been raised they are required to refer these to the Lead Assessment Writer.

It must be stressed that a mark scheme is a working document, in many cases further developed and expanded on the basis of students' reactions to a particular paper. Assumptions about future mark schemes on the basis of one year's document should be avoided; whilst the guiding principles of assessment remain constant, details will change, depending on the content of a particular examination paper.

Further copies of this Mark Scheme are available from [aqa.org.uk](http://aqa.org.uk)

**Key to mark scheme abbreviations**

|              |  |
|--------------|--|
| M            | mark is for method   |
| m or dM      | mark is dependent on one or more M marks and is for method         |
| A            | mark is dependent on M or m marks and is for accuracy              |
| B            | mark is independent of M or m marks and is for method and accuracy |
| E            | mark is for explanation  |
| ✓ or ft or F | follow through from previous incorrect result                      |
| CAO          | correct answer only  |
| CSO          | correct solution only  |
| AWFW         | anything which falls within  |
| AWRT         | anything which rounds to   |
| ACF          | any correct form   |
| AG           | answer given   |
| SC           | special case   |
| OE           | or equivalent  |
| A2,1         | 2 or 1 (or 0) accuracy marks                                       |
| -x EE        | deduct x marks for each error                                      |
| NMS          | no method shown  |
| PI           | possibly implied   |
| SCA          | substantially correct approach                                     |
| c            | candidate  |
| sf           | significant figure(s)  |
| dp           | decimal place(s)   |

**No Method Shown**

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

**Otherwise we require evidence of a correct method for any marks to be awarded.**

| Q            | Solution  | Mark                           | Total     | Comment  |
|--------------|---|--------------------------------|-----------|--|
| 1 (a) (i)    | $\mathbf{a} = \frac{dv}{dt}$<br>$\mathbf{a} = -8 \sin 2t \mathbf{i} + 3 \cos t \mathbf{j}$<br>Using $\mathbf{F} = m\mathbf{a}$<br>$\mathbf{F} = 4 \times \{-8 \sin 2t \mathbf{i} + 3 \cos t \mathbf{j}\}$<br>$= -32 \sin 2t \mathbf{i} + 12 \cos t \mathbf{j}$                      | B1<br>M1<br>A1                 | 3         | All correct<br>Multiplying their a by 4 [must be a vector with at least one trig term]<br>CAO  |
| (ii)         | When $t = \pi$ , $\mathbf{F} = -12 \mathbf{j}$<br>Magnitude of $\mathbf{F}$ is 12   | B1<br>B1                       | 2         | CAO<br>CAO   |
| (b)          | $\mathbf{r} = 2 \sin 2t \mathbf{i} - 3 \cos t \mathbf{j} + \mathbf{c}$<br><br>When $t = 0$ , $\mathbf{r} = 2\mathbf{i} - 14\mathbf{j}$ ,<br>$\therefore \mathbf{c} = 2\mathbf{i} - 11\mathbf{j}$<br>$\therefore \mathbf{r} = (2 \sin 2t + 2)\mathbf{i} - (3 \cos t + 11)\mathbf{j}$ | M1<br>A1<br><br>m1<br>A1<br>A1 | 5         | M1 one term correct<br>A1 another term correct<br>Condone lack of + c<br><br>m1 use of + c [c $\neq$ 0]<br>A1 CAO<br>CAO [accept uncollected form and ISW [condone lack of brackets but must have - 11j] |
| <b>Total</b> |   |                                | <b>10</b> |  |

| Q            | Solution   | Mark                         | Total    | Comment  |
|--------------|--|------------------------------|----------|--|
| 2            | Resolve vertically<br>$R = 3g + 4g + 5g + 8g$<br>$R = 20g$<br><br>Taking moments about A<br>$3 \times 4g + AC \times 8g + 6 \times 5g = 4.3 \times 20g$<br><br>$42g + AC \times 8g = 86g$<br><br>$AC = \frac{44}{8}$<br>Distance AC is 5.5 m | B1<br><br>M1<br>A1<br><br>A1 | 4        | $\bar{x} \sum m_i = \sum x_i m_i$<br>$\sum m_i = 20$ B1<br><br>or moments about any point need 4 non zero terms; could have 20 incorrect<br>all terms either with/without g<br>A1 for all terms correct<br><br>CAO |
| <b>Total</b> |  |                              | <b>4</b> |  |

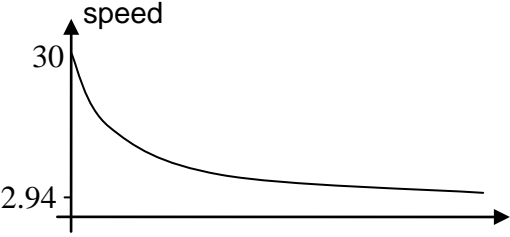
| Q                | Solution  | Mark                             | Total    | Comment  |
|------------------|---|----------------------------------|----------|--|
| <b>3 (a) (i)</b> | P is 2 metres above QR.<br><br>KE = change in PE<br>$= mgh = 32 \times 9.8 \times 2$<br>$= 64 \text{ g or } 627.2 \text{ J}$<br>$= 627 \text{ J}$   | B1<br><br>M1<br>A1               | <b>3</b> | Do not accept unsimplified expression<br><br>Correct terms, any value of h used<br>CAO<br>AWRT   |
| <b>(ii)</b>      | Speed of Simon is $\sqrt{\frac{627.2}{\frac{1}{2} \times 32}}$<br>$= 6.26 \text{ ms}^{-1}$  | M1<br><br>A1                     | <b>2</b> | Ft from their a<br><br>CAO [AWRT]<br>Accept square root 4g or 2 root g   |
| <b>(b)</b>       | Work done travelling Q to R is $F \times 5$<br><br>$R = 32 \text{ g}$<br><br>Work done = change in energy<br>$\mu \times 32\text{g} \times 5 = 64 \text{ g or } 627.2$<br><br>$\mu = 0.4$ | B1<br><br>B1<br><br>M1<br><br>A1 | <b>4</b> | Needs F times 5<br><br>CAO [or 313.6]<br><br>Ft their 32g and their 64g [from a]<br>condone incorrect distance [eg, 7, 9, 4, 2]<br>CAO<br>Or<br>if constant acceleration;<br>B1 for 32 g<br>B1for acceleration = $\pm 2\text{g}/5$ or $\pm 3.92$<br>M1 for $\mu\text{g} = 2\text{g}/5$<br>A1 for 0.4 |
|                  | <b>Total</b>  |                                  | <b>9</b> |  |

| Q     | Solution   | Mark                     | Total            | Comment  |
|-------|--|--------------------------|------------------|--|
| 4 (a) | Resolve vertically<br>$T_{AP} \cos 20 = 5g$<br><br>$T_{AP} = 52.1 \text{ N}$   | M1A1<br><br>A1           | <br><br><b>3</b> | M1 could be sin 20<br>A1 correct<br><br>CAO AWRT   |
| (b)   | Resolve horizontally<br>$T_{AP} \sin 20 + T_{BP} = m \frac{v^2}{r}$<br><br>$T_{BP} = 5 \frac{v^2}{0.6} - \frac{5g}{\cos 20} \sin 20$<br>$= \frac{25}{3} v^2 - 5g \tan 20$<br><br><b>AG</b> | M1<br>A1<br><br>A1       | <br><br><b>3</b> | Needs all the terms, could be cos 20<br>Needs sin 20 or cos 70                           |
| (c)   | $T_{AP} = T_{BP}$<br>$\frac{25}{3} v^2 - 5g \tan 20 = 52.1 \text{ or } \frac{5g}{\cos 20^\circ}$<br><br>$\frac{25}{3} v^2 = 69.9$<br>$v^2 = 8.388 \text{ or } 8.3975$<br>$v = 2.90$        | M1A1<br><br>A1<br><br>A1 | <br><br><b>4</b> | ft from (a)<br><br><b>CAO PI</b><br><br>Or 2.896.. or 2.8978 CAO<br><br>2.9 not accepted |
|       |  |                          |                  |  |
|       | <b>Total</b>   |                          | <b>10</b>        |  |

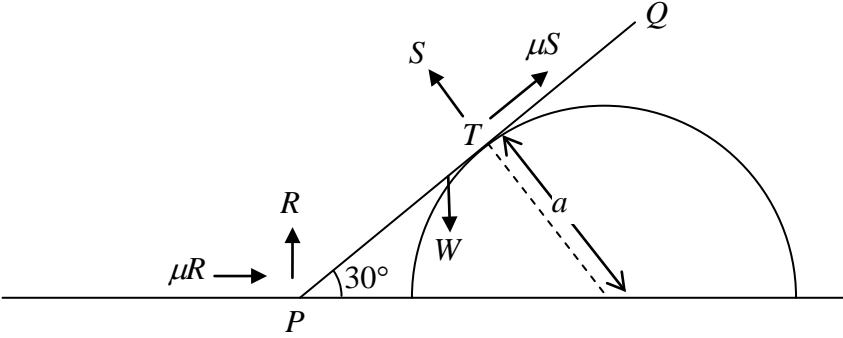
| Q | Solution   | Mark   | Total    | Comment  |
|---|--|--|----------|--|
| 5 | 900 rpm<br>$= 900 \times \frac{2\pi}{60}$ radians per second<br>$= 30\pi$ radians per second<br><br>Minimum reactive force is<br>$m\omega^2 r - mg$<br>$= 0.8 \times (30\pi)^2 \times 0.3 - 0.8g$<br>$= 2131.83 - 7.84$<br>Minimum magnitude is 2123.99<br>$= 2120$<br><br>Maximum reactive force is<br>$m\omega^2 r + mg$<br>$= 0.8 \times (30\pi)^2 \times 0.3 + 0.8g$<br>$= 2131.83 + 7.84$<br>Maximum magnitude is 2139.67<br>$= 2140$ | M1<br>A1<br><br>M1<br><br>A1<br><br>M1<br><br>A1 | 6        | Use of $\frac{2\pi}{60}$<br>CAO or for $v = 9\pi$ or 28.27 or 28.3<br><br>Needs both terms and correct signs<br>could be using $v$<br><br>CAO AWRT<br><br>Needs both terms and correct signs<br><br>CAO AWRT<br>[must be clear which is min/max<br>unless in this order] |
|   | <b>Total</b>   |  | <b>6</b> |  |





| Q            | Solution   | Mark                                   | Total    | Comment   |
|--------------|--|--|----------|---|
| <b>7(a);</b> | Using $F = ma$<br>$72 \frac{dv}{dt} = 72g - 240v$<br>$-\frac{3}{10} \frac{dv}{dt} = v - 2.94$  | M1<br><br>A1                           | 2        | CAO<br><br>AG; Needs M1 above   |
| <b>(b)</b>   | Hence $\int \frac{1}{v-2.94} dv = -\frac{10}{3} \int dt$<br>$\ln(v-2.94) = -\frac{10}{3}t + c$<br><br>$v - 2.94 = Ce^{-\frac{10}{3}t}$<br>$t = 0, v = 30$<br>$\therefore C = 27.06$<br><br>$\therefore v = 2.94 + 27.06e^{-\frac{10}{3}t}$ | M1A1<br><br>m1<br><br><br>A1<br><br>A1 | 5        | M1 for either side integrated correctly<br>A1 for all correct<br>m1 for + c<br><br><br>CAO condone 1353/50<br>accept $c = \ln 27.06$<br><br>CAO condone 27.1m |
| <b>(c)</b>   |   | B2                                     | 2        | B1 for starting at 30 and basic shape<br>B1 for asymptote of 2.94   |
|              | <b>Total</b>   |  | <b>9</b> |   |

| Q             | Solution   | Mark   | Total     | Comment   |
|---------------|--|--|-----------|---|
| <b>8 (a)</b>  | When $x \geq 26$ ,<br>KE is $\frac{1}{2} \times 70 \times v^2$<br>EPE is $\frac{1456 \times (x-26)^2}{2 \times 26}$<br>Change in PE is $70 \times g \times x$<br><br>Conservation of energy :<br>$\frac{1}{2} \times 70 \times v^2 + \frac{1456 \times (x-26)^2}{2 \times 26} = 70 g \times x$<br><br>$35v^2 + 28(x-26)^2 = 70gx$<br>$5v^2 + 4(x-26)^2 = 98x$<br>$5v^2 = 306x - 4x^2 - 2704$ | M1A1<br>A1<br><br><br><br><br><br><br><br><br><br>A1 | 4         | M1 for 3 terms of correct items<br>A1 for 2 of the 3 types of energy are correct [ignore signs]<br>[treat all GPE terms as one term]<br><br>A1 for all terms correct [70g is 686]<br>Accept 4 terms if PE is on both sides<br><br>CAO |
| <b>(b)</b>    | If $x$ is not greater than 26, cord is not stretched.<br><br>Hence EPE cannot be used unless $x$ is greater than 26.   | B1   | 1         | Either statement,<br>or cord not taut<br>no EPE   |
| <b>(c)</b>    | At maximum value of $x$ , $v = 0$<br>$\therefore 4x^2 - 306x + 2704 = 0$<br>$x = 66.3$   | M1<br><br>A1   | 2         | Correct use of $v = 0$<br><br>CAO [bod if give 2 values]  |
| <b>(d)(i)</b> | When speed is a maximum, $a = 0$<br>tension = gravitational force<br><br>$\frac{1456 \times (x-26)}{26} = 70g$<br>$x - 26 = 12.25$<br>$x = 38.25$  | M1<br><br><br>A1                                     | 2         | or differentiating (a)<br>$306 - 8x = 0$<br><br>Accept 38.2 or 38.3<br>Could be seen with no working  |
| <b>(ii)</b>   | Using (a) and (d)(i)<br>for maximum speed<br>$5v^2 = 11704.5 - 5852.25 - 2704$<br>$v^2 = 629.65$<br>Maximum speed is $25.1 \text{ ms}^{-1}$  | B1   | 1         | CAO   |
|               | <b>Total</b>   |  | <b>10</b> |   |

| Q | Solution  | Mark  | Total    | Comment  |
|---|---|---|----------|--|
| 9 |   |   |          |  |
|   | $a / PT = \tan 30$ $PT = \frac{a}{\tan 30}$ <p>Resolve vertically</p> $R + S \cos 30 + \mu S \sin 30 = W \quad (1)$ <p>Resolve horizontally</p> $\mu R + \mu S \cos 30 = S \sin 30 \quad (2)$ <p>Moments about P</p> $PT \times S = W \times a \cos 30$ $\frac{a \cos 30}{\sin 30} \times S = W \times a \cos 30$ $S = W \sin 30 \quad \text{or} = \frac{1}{2} W$ <p>(2) <math>\rightarrow \mu R = W (\sin^2 30 - \mu \sin 30 \cos 30)</math><br/>             or <math>\mu R = W (\frac{1}{4} - \frac{\sqrt{3}}{4} \mu)</math></p> <p>(1) <math>\rightarrow \mu R + \mu S \cos 30 + \mu^2 S \sin 30 = \mu W</math><br/> <math>W (\frac{1}{4} - \frac{\sqrt{3}}{4} \mu) + \mu \frac{1}{2} W \frac{\sqrt{3}}{2} + \mu^2 \frac{1}{2} W \frac{1}{2} = \mu W</math><br/> <math>\mu = \frac{\sin^2 30 + \mu^2 \sin^2 30}{\cos 30 - \mu \sin 30}</math><br/>             or <math>\mu = \frac{1}{4} + \frac{1}{4} \mu^2</math><br/> <math>\mu^2 - 4\mu + 1 = 0</math><br/> <math>\mu = 2 - \sqrt{3}</math> or 0.268</p> | <p>B1</p> <p>M1A1</p> <p>M1A1</p> <p>B1</p> <p>m1</p> <p>A1</p> | <p>8</p> | <p><b>Or</b> resolve along the rod<br/> <math>\mu S + R \sin 30 + \mu R \cos 30 = W \sin 30</math><br/>                 M1 for any 4 terms; must include at least 1 friction term and a trig term</p> <p>Resolve perpendicular to rod<br/> <math>S + R \cos 30 = \mu R \sin 30 + W \cos 30</math><br/>                 M1 for any 4 terms; must include at least 1 friction term and a trig term</p> <p>If resolve horizontally M1 for any 3 terms; must include a trig term</p> <p>Allow, bod, if moments taken about another point</p> <p><math>R(\sin 30 + \mu \cos 30) = W \sin 30 (1 - \mu)</math></p> <p><math>R(\cos 30 - \mu \sin 30) = W(\cos 30 - \sin 30)</math><br/>                 Dividing<br/> <math>\frac{\sin 30 + \mu \cos 30}{\cos 30 - \mu \sin 30} = \frac{\sin 30(1 - \mu)}{\cos 30 - \sin 30}</math><br/>                 m1 for simplifying into a quadratic<br/>                 Dependent on both M1 above<br/>                 condone <math>\mu = 2 + \sqrt{3}</math></p> |
|   | <b>Total</b>  |   | <b>8</b> |  |

